***10:40 – 12:20***

**FA4 Networks II**

*Contributed Session*

*Chair:* Dalila B. M. M. Fontes *Room:* 6.2.47

## 1 - Robustness assessment of complex networks based on the Kirchhoff index

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**Abstract**

This paper is aimed to the inspection of a graph measure called effective graph resistance, also known as Kirchhoff index (or resistance distance), derived from the field of electric circuit analysis. It is defined as the accumulated effective resistance between all pairs of vertices. This index is widely used in Mathematical Chemistry, Computational Biology and, more generally in Network Analysis in order to describe the graph topology. The objective of the paper is twofold. First, we survey known results regarding the Kirchhoff index and we discuss a methodology in order to obtain some new and tighter bounds of this graph invariant. The derivation of these new limitations takes advantage of real analysis techniques, based on majorization theory and optimization of functions which preserve the majorization order, the so-called Schur-convex functions. Secondly, we focus on the application of this topological index in the analysis of robustness-related problems. It is worth pointing out that the Kirchhoff index can be highly valuable and informative as a robustness measure of a network, showing the ability of a network to continue performing well when it is subject to failure and/or attack. In fact, the pairwise effective resistance measures the vulnerability of a connection between a pair of vertices that considers both the number of paths between the vertices and their length. A small value of the effective graph resistance therefore indicates a robust network. Being the calculation of the exact value of the Kirchhoff index computationally intensive, bounds on this graph invariant have been also proposed in the literature as an alternative measure of robustness. In particular, the fact that the Kirchhoff index can be also expressed by the Laplacian eigenvalues, entails a relation with the algebraic connectivity, that is often applied as a useful approximation to assess robustness. However, it has been shown that the algebraic connectivity may not display desirable properties for a robustness indicator. Within this topological robustness framework, we propose to use our bounds, obtained via majorization techniques, for robustness assessment of complex networks. A comparison with alternative graph measures is provided by applying our methodology to random network models and real networks. Further research could regard a generalization to weighted and/or directed networks and the analysis of the correlation between alternative topological metrics.

## 2 - Optimization models and methods for "Rota do Românico"

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**Abstract**

In this work the real case of the "Rota do Românico" (www.rotadoromanico.com/) is presented. At the region of north of Portugal, more precisely at the Tâmega and Sousa region, the tourist that visits this region may visit a vast number of Romanic monuments in the region. "Rota do Românico" presents routes from Vale do Tâmega and Vale do Douro for visiting these monuments. Data regarding the locations of all the monuments, and also all possible roads and motorways connecting these monuments, were collected. In this presentation we will explore several linear and nonlinear optimization models for determining the best route that a traveler should take for visiting a set of monuments of its choice, by choosing different criterions. This problem is a variation of the well-known traveler salesman problem. We will present computational results obtained using several optimization techniques for solving real instances from the "Rota do Românico" real case.

## 3 - Locating a cluster head for minimum-power under symmetric range assignment

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**Abstract**

For a star network consisting of a given set of nodes on a Euclidean plane and a master node at the star point, the object is to optimise the location of the star point. This unconstrained convex optimisation is with respect to the power required by the network. The power required by a node is a quadratic function of its distance to the master node, and for the master node is a quadratic function of its distance to a farthest node. The power required by the network is the sum of the requirements of the nodes and the master node. The optimised location of the star point, that which minimises the power requirement of the network, is defined as the min-power centre. The sum of the quadratic functions is a strictly convex function, which ensures that there is only one min-power centre. The optimisation process begins at the centroid of the given set of nodes, where we establish a star point that we will move in the direction that produces the maximum rate of reduction of the value of the power function. It turns out that the optimisation path of the star point that provides the maximum rate of decrease in the value of the power function is a series chain of straight edges which lie on the perpendicular bisectors of edges joining particular pairs of nodes. The path terminates at the min-power centre, which is the point of convexity of the power function, or is the centre of a circle on which lie three nodes, including a node farthest from the centroid, that form an acute angled triangle. In the latter case no further decrease is possible. Algebraic characterisations of the possible min-power centres for different given sets of nodes, and associated values of the power function, are provided.

## 4 - Heuristics solutions for the maximum edge weight clique problem: a quadratic approach

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**Abstract**

This work addresses the maximum edge weight clique problem, a generalization of the well-known maximum clique problem.
We propose to address this problem by resorting to a quadratic discrete formulation. This is then converted into an equivalent quadratic continuous formulation, from which a heuristic approach is derived based on the optimization of a quadratic function over a sphere.
Preliminary computational results are reported for a subset of benchmark problem instances derived from the DIMACS maximum clique instances.

Acknowledgments: We acknowledge the financial support of "NORTE-01-0145-FEDER-000020", financed by the North Portugal Regional Operational Programme (NORTE 2020), under the PORTUGAL 2020 Partnership Agreement.